Valovanje na vodni gladini

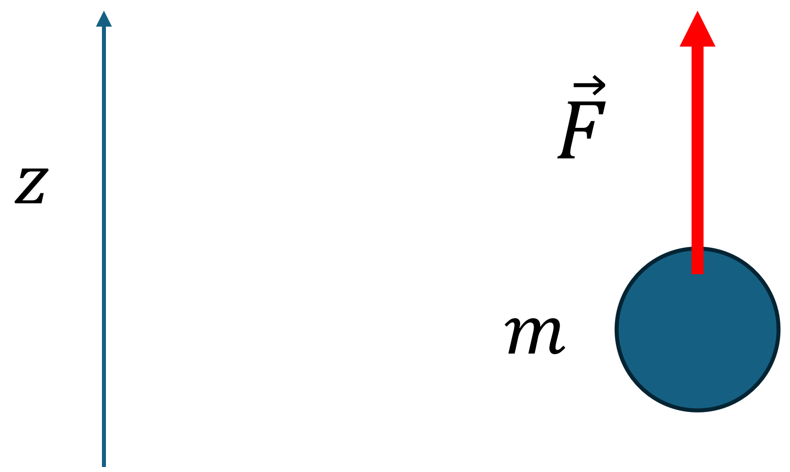
fizikalni članek, Presek 52, 1

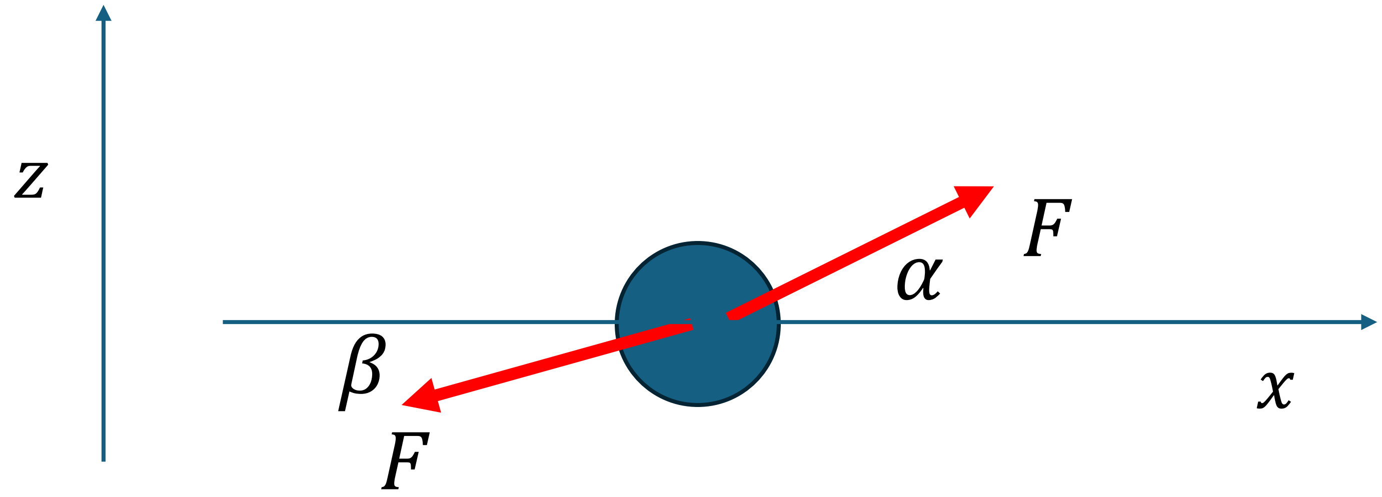
Zakaj vodni valovi?

- proučevanje vodnih površinskih valov je bogata in zanimiva tema za dijake
- vsakdanji pojav
- obravnava je interdisciplinarna, povezuje matematiko, fiziko in vede o okolju - oceanografija, inženirstvo in klimatologija

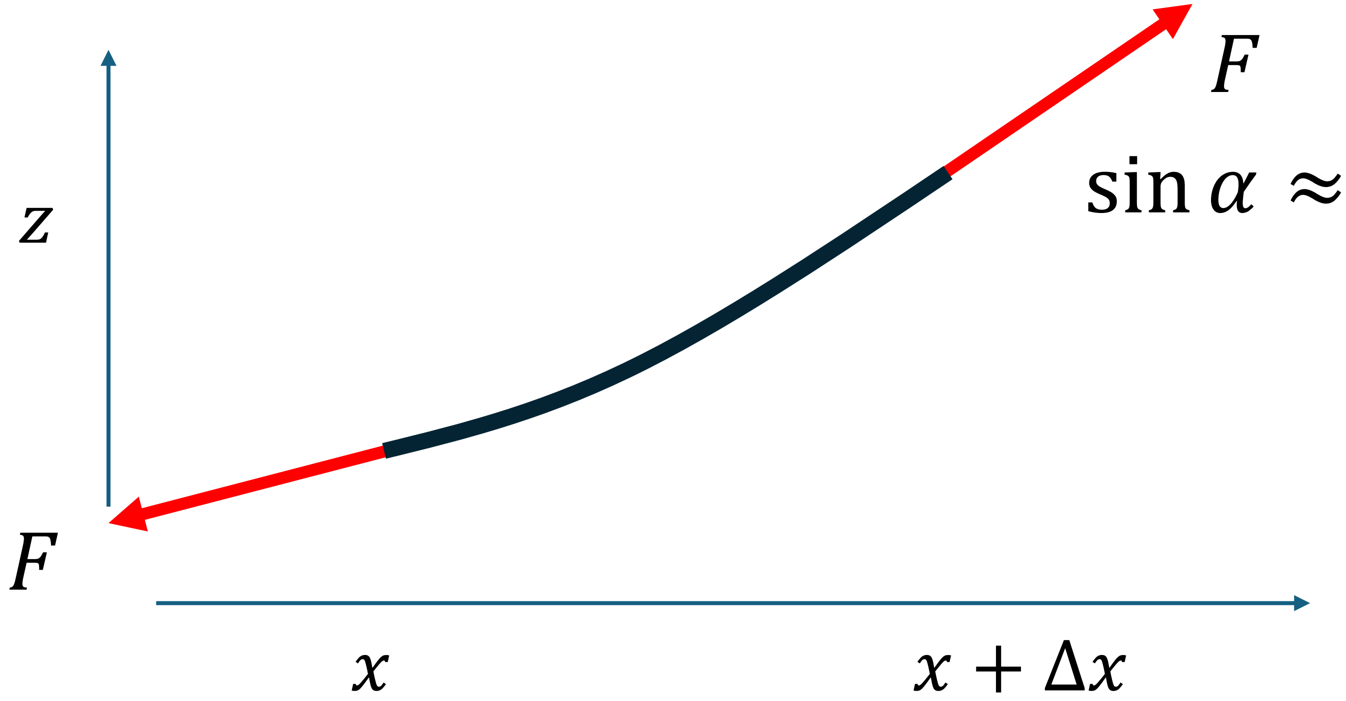
Newtonov zakon

- osnovna dinamična enačba mehanike
- $\vec{F} = m\,\vec{a}$
- diferencialna enačba drugega reda: $a_z = \frac{d^2z}{dt^2} = \frac{1}{m} F_z$





$$\ddot{z} = \frac{1}{m} F(\sin\alpha - \sin\beta)$$



$$\sin\alpha \approx \alpha \approx \tan\alpha = \frac{\Delta z}{\Delta x}$$

$$\ddot{z} = \frac{1}{\mu\Delta x} F\left(z'(x+\Delta x) - z'(x)\right) = c^2 z''$$

valovna enačba

- majhni odmiki
- zvezni medij

- $\ddot{z} = c^2 z''$

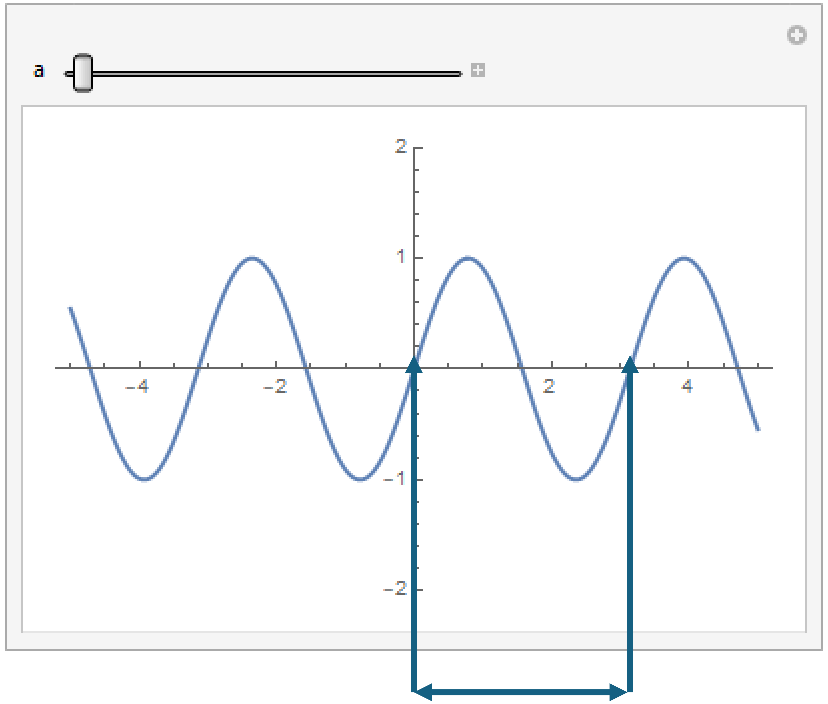
- parcialna diferencialna enačba drugega reda

- splošna rešitev $z = u(x - ct)$

harmonično valovanje, lastna funkcija valovne enačbe $c^2 u'' = \ddot{u}$

- $u = u_0 \sin(kx - \omega t)$

- hitrost $c = \frac{\lambda}{t_0} = \lambda \nu = \frac{\omega}{k}$
- frekvenca $\omega = 2\pi\nu$
- valovna dolžina $k = \frac{2\pi}{\lambda}$
- amplituda



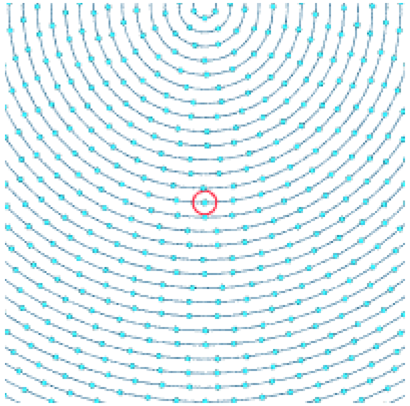
valovna dolžina λ

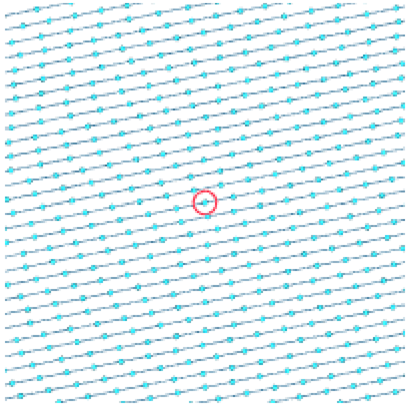
vodni površinski valovi

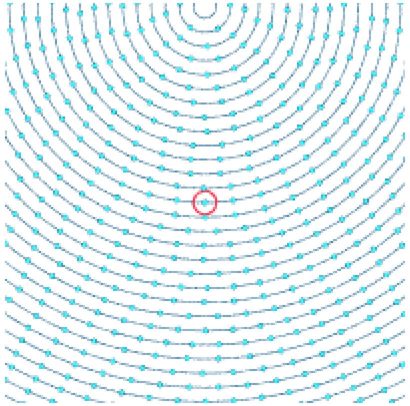
- motnje, ki se širijo vzdolž meje med vodo in zrakom
- različne vrste vodnih valov:
  - kapilarni valovi (majhni valovi)
  - gravitacijski valovi (večji valovi).

Airyeva teorija

- neviskozna, nestisljiva (ρ = konst.) tekočina, irotacionalni (brezvrtinčni) tok



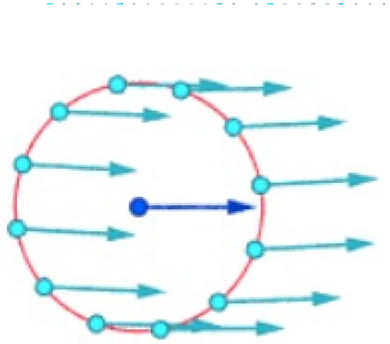


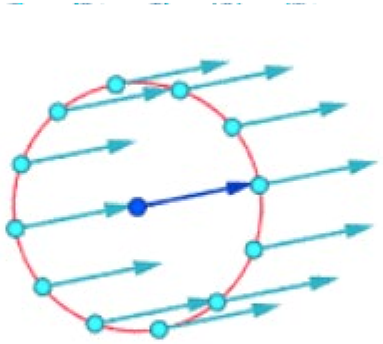


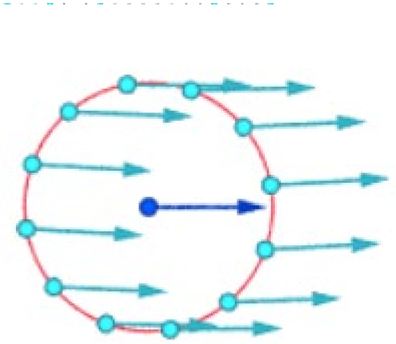
- v~r   strižni tok   v~1/r

Airyeva teorija

- neviskozna, nestisljiva (ρ = konst.) tekočina, irotacionalni (brezvrtinčni) tok



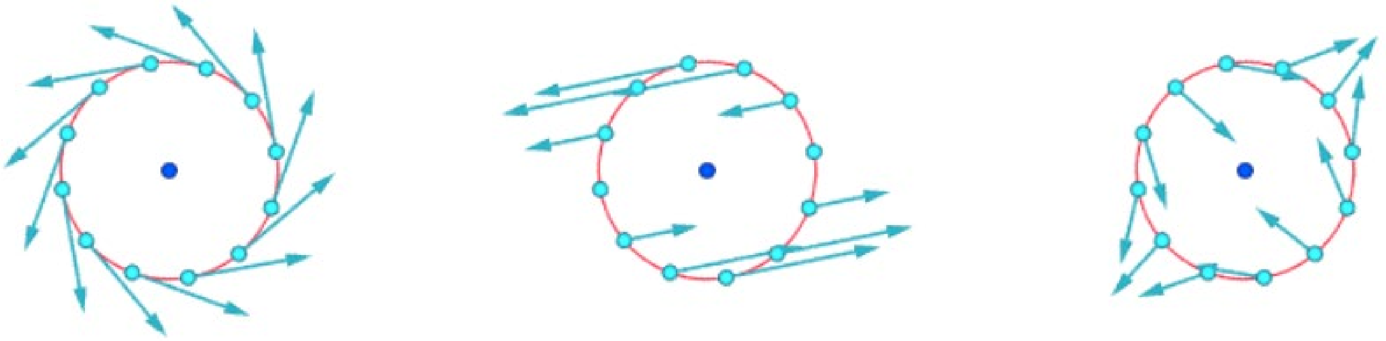




- v~r strižni tok v~1/r

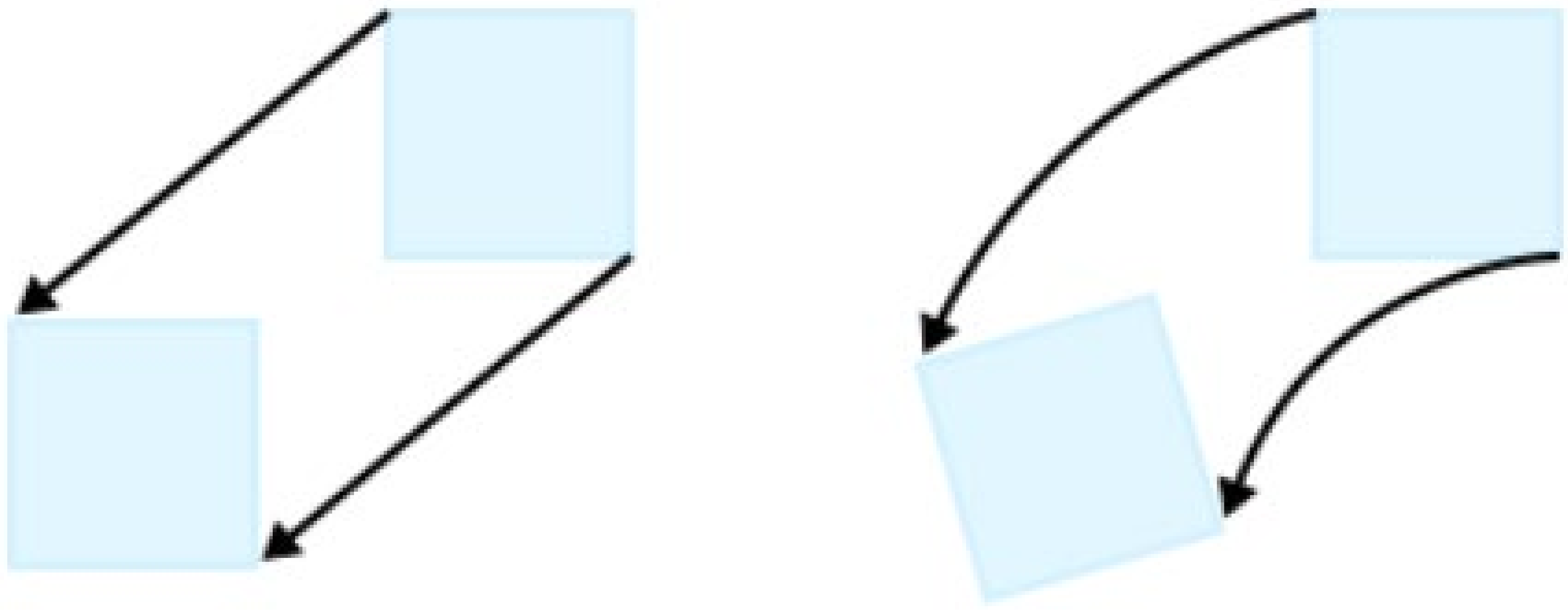
Airyeva teorija

- neviskozna, nestisljiva (ρ = konst.) tekočina, irotacionalni (brezvrtinčni) tok



Vorticity ≠ 0 | Vorticity ≠ 0 | Vorticity = 0

- v~r strižni tok v~1/r



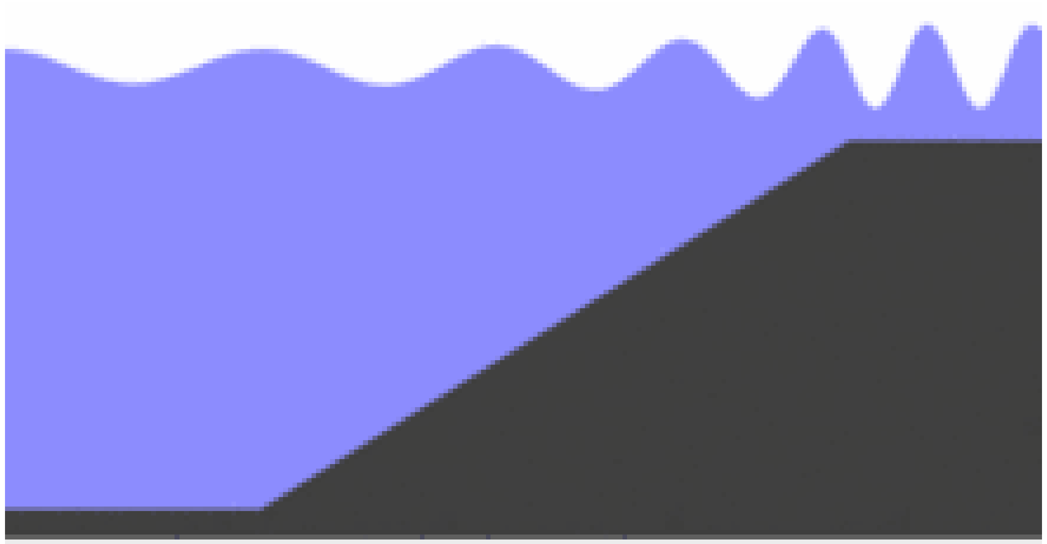
potencialni tok

- gladina se nahaja pri $z = u(x,t)$, dno pri $z = -h$
- $v_x = \frac{\partial \phi}{\partial x}$
- $v_z = \frac{\partial \phi}{\partial z}$
- skupaj s kontinuitetno enačbo $\nabla \cdot \vec{v} = 0$ sledi
- $\nabla^2 \phi = 0$
- robni pogoj v_z na dnu = 0
- na vrhu pa je $v_z = \frac{\partial \phi}{\partial z} = u(x,t)$
- linearizacija Bernoullijeve enačba ($p + \rho g z + \frac{1}{2}\rho v^2$=konst.) da dinamični pogoj (prosta površina s tlakom 0)
- $\frac{\partial \phi}{\partial t} + g\, u = 0$ pri $z = u$

- za val $u = u_0 \cos(kx - \omega t)$
- je rešitev za potencial
- $\phi = \frac{\omega}{k} u_0 \frac{\cosh k(z+h)}{\sinh kh} \sin(kx - \omega t)$
- dinamičnemu robnemu pogoju je zadoščeno pri disperzijski zvezi
- $\omega^2 = gk \tanh kh$
- v globoki vodi je $\omega = \sqrt{gk}$
- disperzija, dolgi valovi so hitrejši: $c = \frac{\omega}{k} = \sqrt{\frac{g}{k}} \propto \sqrt{\lambda}$
- v plitvi vodi ni disperzije $\omega = \sqrt{gh}\, k$ in $c = \sqrt{gh}$

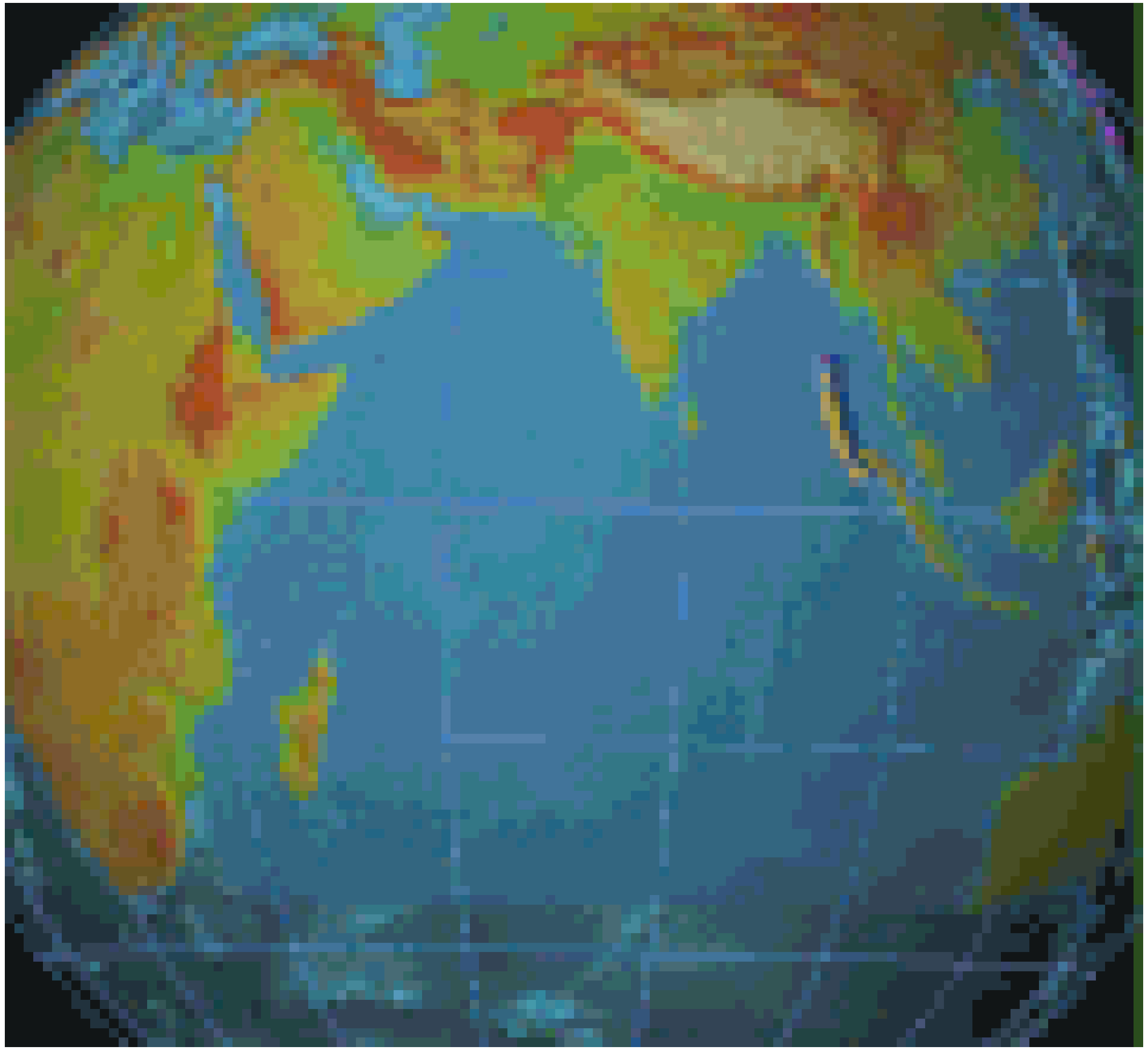
cunami

- valovna dolžina cunamija ~ 200 km
- povprečna globina Pacifika je 4 km
- hitrost valovanja 200 m/s (800 km/h)
- perioda 20 min
- amplituda 1 m
- moč valovanja $P \propto u_0^2 \omega^2 c$
- $u_{01}^2 c_1 = u_{02}^2 c_2$
- Greenov zakon $u_{01}^4 h_1 = u_{02}^4 h_2$
- amplituda na globini 10 m je 4 m



$$c = \sqrt{gh}$$

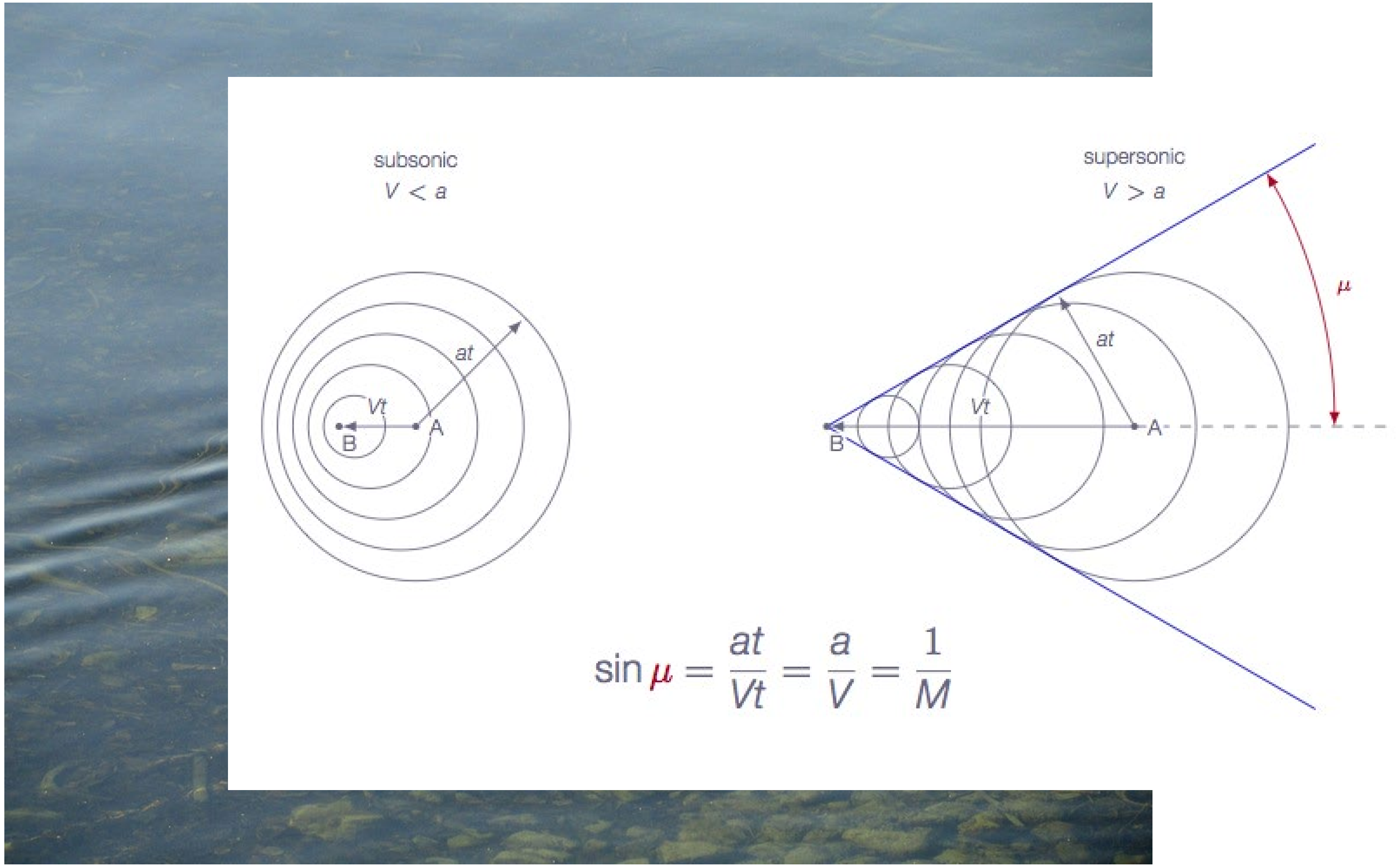
cunami 2004



kapilarni val

kelvinova brazda

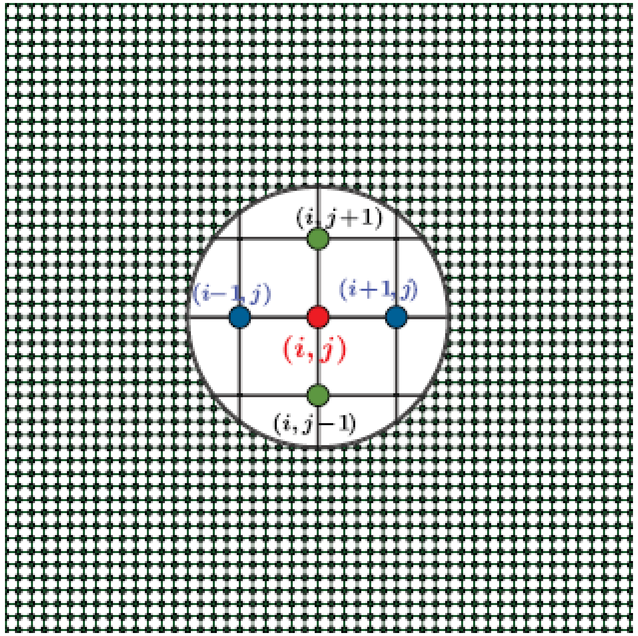
kelvinova brazda – ni Machovo valovno čelo

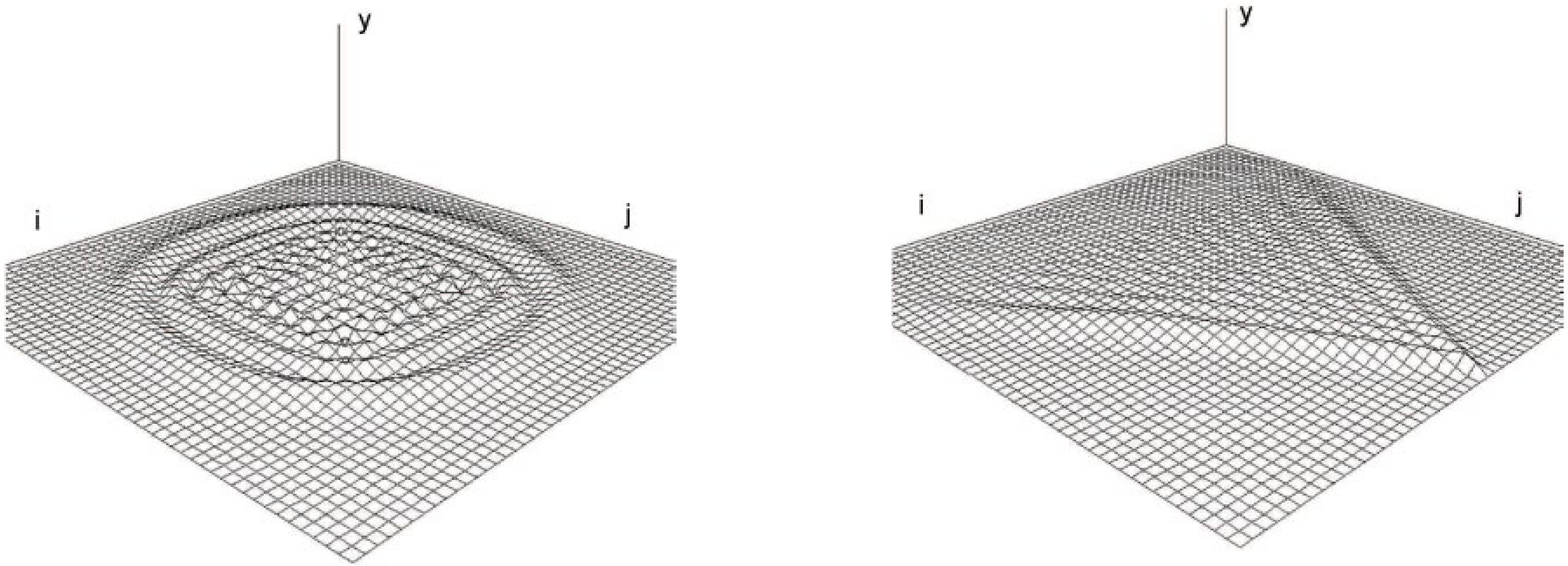


mreža povezanih kroglic

- diferenčna enačba za veliko množico nihal

$$y_{i,j}^{m} = 2y_{i,j}^{m-1} - y_{i,j}^{m-2} + \frac{F_0\Delta t^2}{Md}[(y_{i-1,j}^{m-1} - 2y_{i,j}^{m-1} + y_{i+1,j}^{m-1}) + (y_{i,j-1}^{m-1} - 2y_{i,j}^{m-1} + y_{i,j+1}^{m-1})].$$

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