Dvojno nihalo po domače

Aleš Mohorič, Univerza v Ljubljani

fizikalni članek, Presek 52, 4

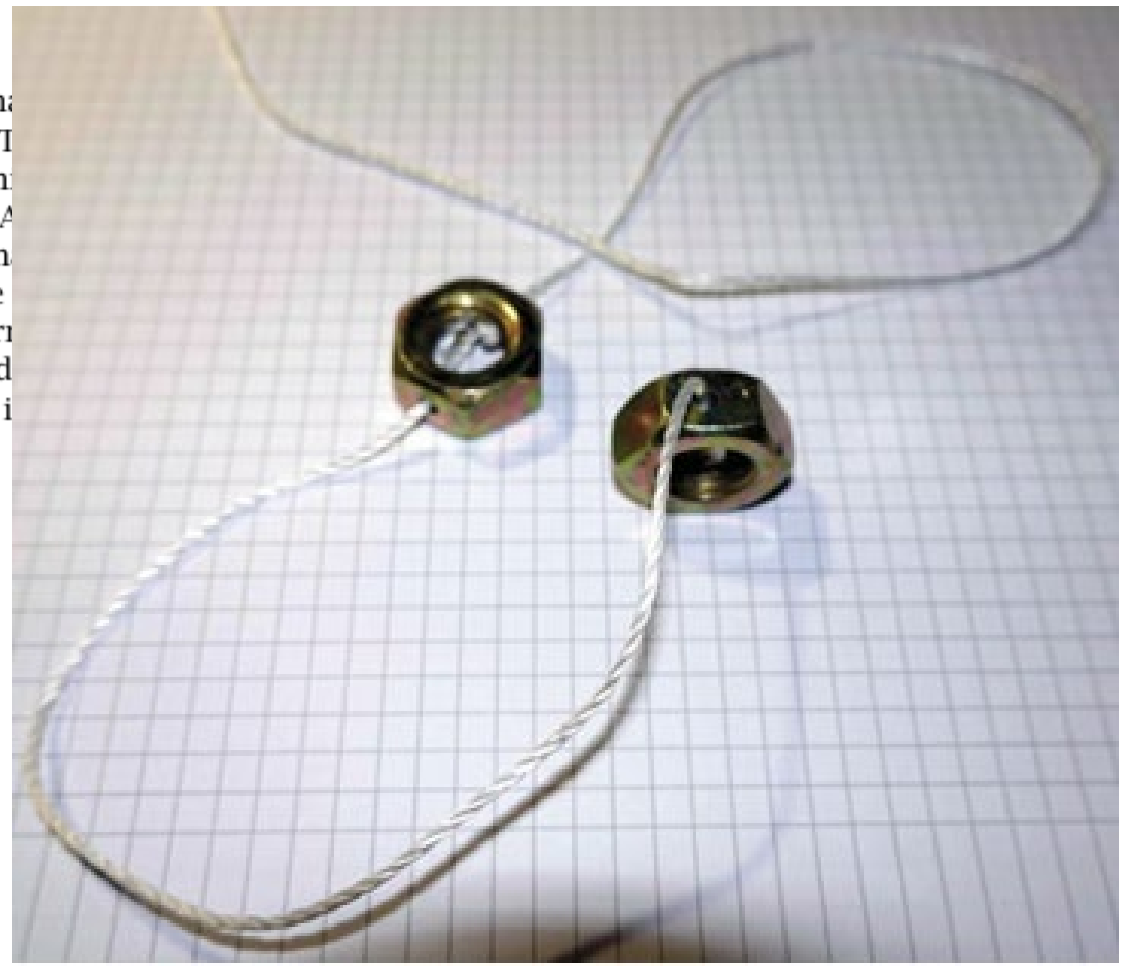
2025

Dvojno nihalo po domače

↓↓↓
Andrej Likar

→ Dvojno nihalo tvorita dve običajni nihali, povezani, kot bi rekli po električno, zaporedno. Na utež, ki je z zelo lahko in togo palico pritrjena na strop, je prav s tako palico pritrjena druga utež, glej sliko 1. Obe palici sta na uteži in strop pritrjeni vrtljivo brez trenja.

zelo zapleteno. Zato se na
zelo majhnimi odmiki. T
nič preprosto in skoraj ni
dalo računsko opisati. A
Poskusimo najti tako njih
zimo pri enojnem, to je
Vemo, da le-to niha harn
odmik od ravnovesja x_0 d
nusno funkcijo časa. Ko i
imamo:

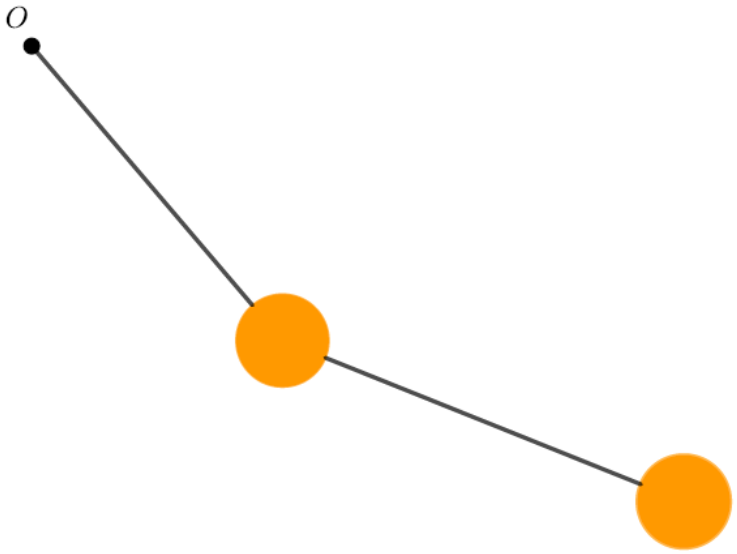


UVOD

- ogledali si bomo dvojno nihalo - preprost, a bogat sistem, ki povezuje fiziko, matematiko in teorijo kaosa.
- je klasičen primer, kako lahko majhne spremembe vodijo do nepredvidljivih rezultatov.
- obravnavali bomo njegovo mehaniko, enačbe, kaos in uporabo

KAJ JE DVOJNO NIHALO?

- sestavljeno je iz dveh (nitnih – fizičnih) nihal, eno je pritrjeno v fiksno osišče, drugo visi na prvem
- sestavni deli: dve masi (m_1, m_2), dve palici (dolžine l_1, l_2), osišče in gravitacija
- za razliko od enega nihala njegovo gibanje ni le periodično



ZGODOVINSKI PREGLED

- 1738 Euler, Bernoulli
- 1788: Lagrangeova mehanika,
- 1890: Poincaréjevi vpogledi v kaos,

Daniel Bernoulli
(1733)

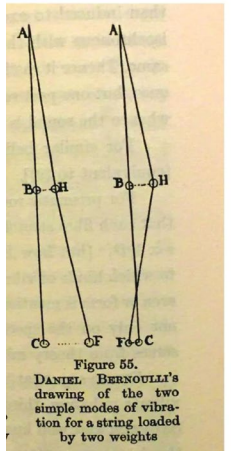


Figure 55.
DANIEL BERNOULLI's drawing of the two simple modes of vibration for a string loaded by two weights

Johann I Bernoulli
(1742)

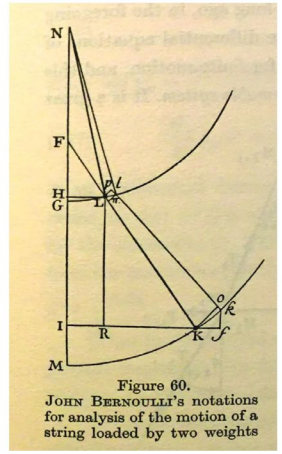
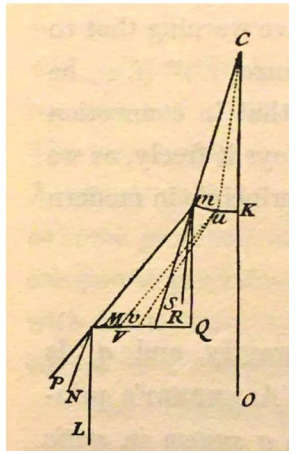
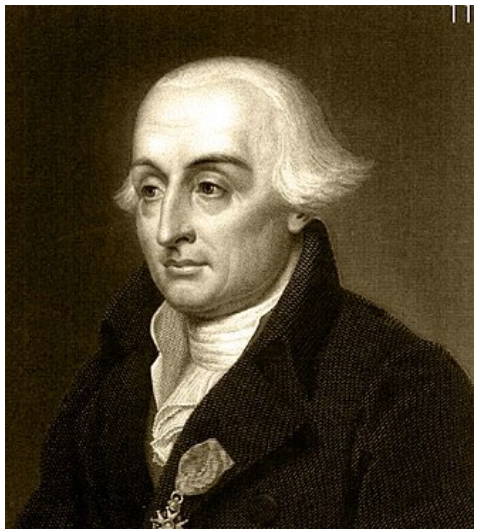


Figure 60.
JOHN BERNOULLI's notations for analysis of the motion of a string loaded by two weights

D'Alembert
(1743)

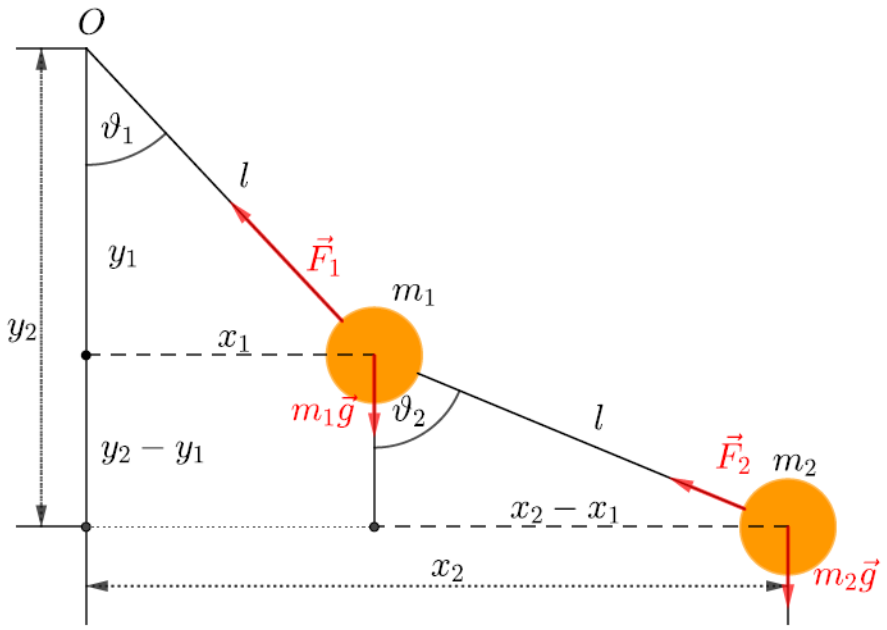




https://galileo-unbound.blog/tag/double-pendulum/

FIZIKA NIHALA

- gravitacija poganja gibanje, medtem ko ga napetost v palicah omejuje
- opišeta ga dva kota (θ_1, θ_2)
- kinetična in potencialna energija se izmenjujeta med nihanjem



LAGRANGEEV FORMALIZEM

Positions

$$x_1 = l_1 \sin\theta_1$$
$$y_1 = -l_1 \cos\theta_1$$
$$x_2 = x_1 + l_2 \sin\theta_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$
$$y_2 = y_1 - l_2 \cos\theta_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$

Velocities

$$\dot{x}_1 = l_1\dot{\theta}_1 \cos\theta_1$$
$$\dot{y}_1 = l_1\dot{\theta}_1 \sin\theta_1$$
$$\dot{x}_2 = \dot{x}_1 + l_2\dot{\theta}_2 \cos\theta_2 = l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2$$
$$\dot{y}_2 = \dot{y}_1 + l_2\dot{\theta}_2 \sin\theta_2 = l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2$$

$$L = \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2) + g(m_1 + m_2)l_1 \cos\theta_1 + gm_2 l_2 \cos\theta_2.$$

https://ir.canterbury.ac.nz/server/api/core/bitstreams/058ff71c-bd7d-4ee5-ad8d-8ada9422995f/content

LAGRANGEEV FORMALIZEM

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,$$

$$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + (m_1 + m_2) g \sin\theta_1 = 0$$

$$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 l_1 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + m_2 g \sin\theta_2 = 0$$

$$\omega_1 = \dot{\theta}_1,$$

$$\omega_2 = \dot{\theta}_2,$$

$$\dot{\omega}_1 = \frac{m_2 l_1 \omega_1^2 \sin(2\Delta\theta) + 2 m_2 l_2 \omega_2^2 \sin\Delta\theta + 2 g m_2 \cos\theta_2 \sin\Delta\theta + 2 g m_1 \sin\theta_1}{-2 l_1 (m_1 + m_2 \sin^2 \Delta\theta)},$$

$$\dot{\omega}_2 = \frac{m_2 l_2 \omega_2^2 \sin(2\Delta\theta) + 2(m_1 + m_2) l_1 \omega_1^2 \sin\Delta\theta + 2 g (m_1 + m_2) \cos\theta_1 \sin\Delta\theta}{2 l_2 (m_1 + m_2 \sin^2 \Delta\theta)}.$$

NEWTONOV FORMALIZEM

$$ma_{x1} = F_{x1} = -F_1 \sin\vartheta_1 + F_2 \sin\vartheta_2 \,,$$

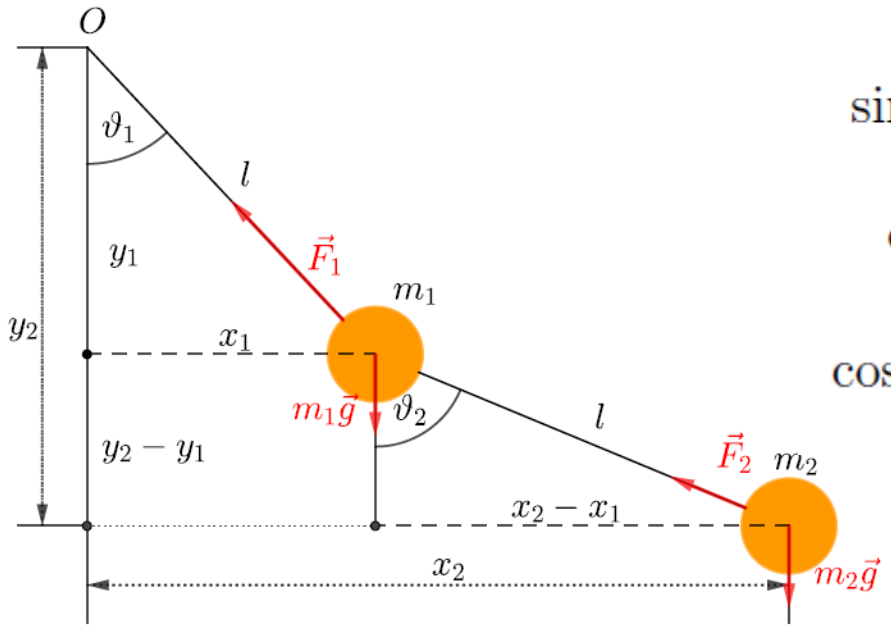
$$ma_{x2} = F_{x2} = -F_2 \sin\vartheta_2 \,,$$

$$ma_{y1} = F_{y1} = -mg + F_1 \cos\vartheta_1 - F_2 \cos\vartheta_2 \,,$$

$$ma_{y2} = F_{y2} = -mg + F_2 \cos\vartheta_2 \,,$$

$$F_1 = k\left(l - \sqrt{x_1^2 + y_1^2}\right) \,,$$

$$F_2 = k\left(l - \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right) \,.$$



$$\sin\vartheta_1 = \frac{x_1}{l} \,,$$

$$\sin\vartheta_2 = \frac{x_2 - x_1}{l} \,,$$

$$\cos\vartheta_1 = -\frac{y_1}{l} \,,$$

$$\cos\vartheta_2 = -\frac{y_2 - y_1}{l} \,.$$

NUMERIČNA SIMULACIJA

$$v_{x1,n+1} = v_{x1,n} + (-f_{1,n} \sin \vartheta_{1,n} + f_{2,n} \sin \vartheta_{2,n})\, \Delta t\,,$$

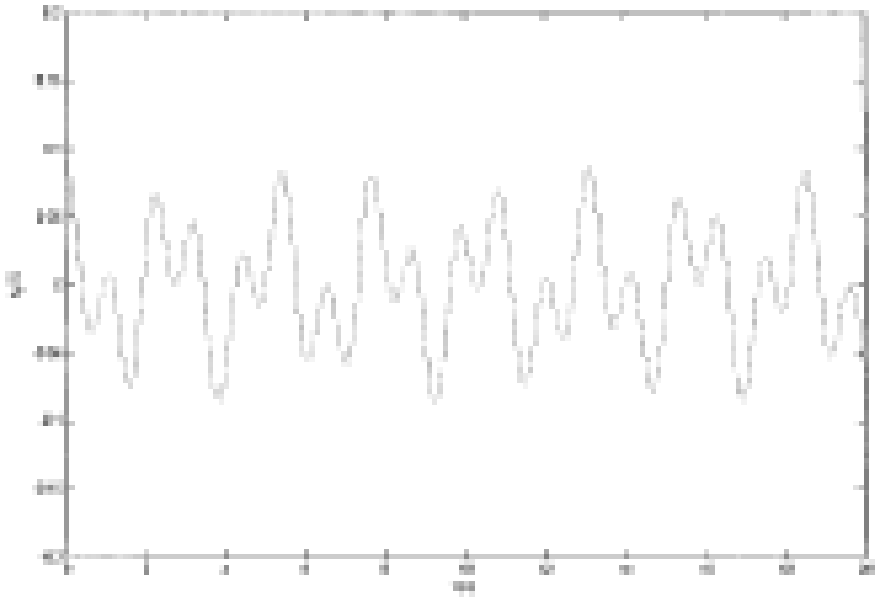
$$v_{y1,n+1} = v_{y1,n} + (-g - f_{1,n} \cos \vartheta_{1,n} - f_{2,n} \cos \vartheta_{2,n})\, \Delta t\,,$$

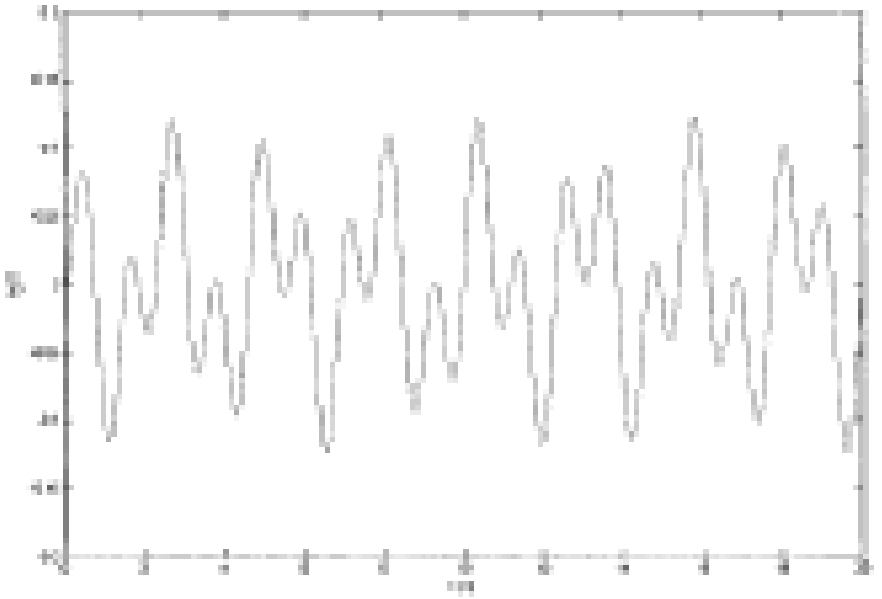
$$v_{x2,n+1} = v_{x2,n} + (-f_{1,n} \sin \vartheta_{2,n})\, \Delta t\,,$$

$$v_{y2,n+1} = v_{y2,n} + (-g + f_{2,n} \cos \vartheta_{2,n})\, \Delta t\,,$$

$$x_{1,n+1} = x_{1,n} + v_{x1,n+1} \Delta t\,.$$

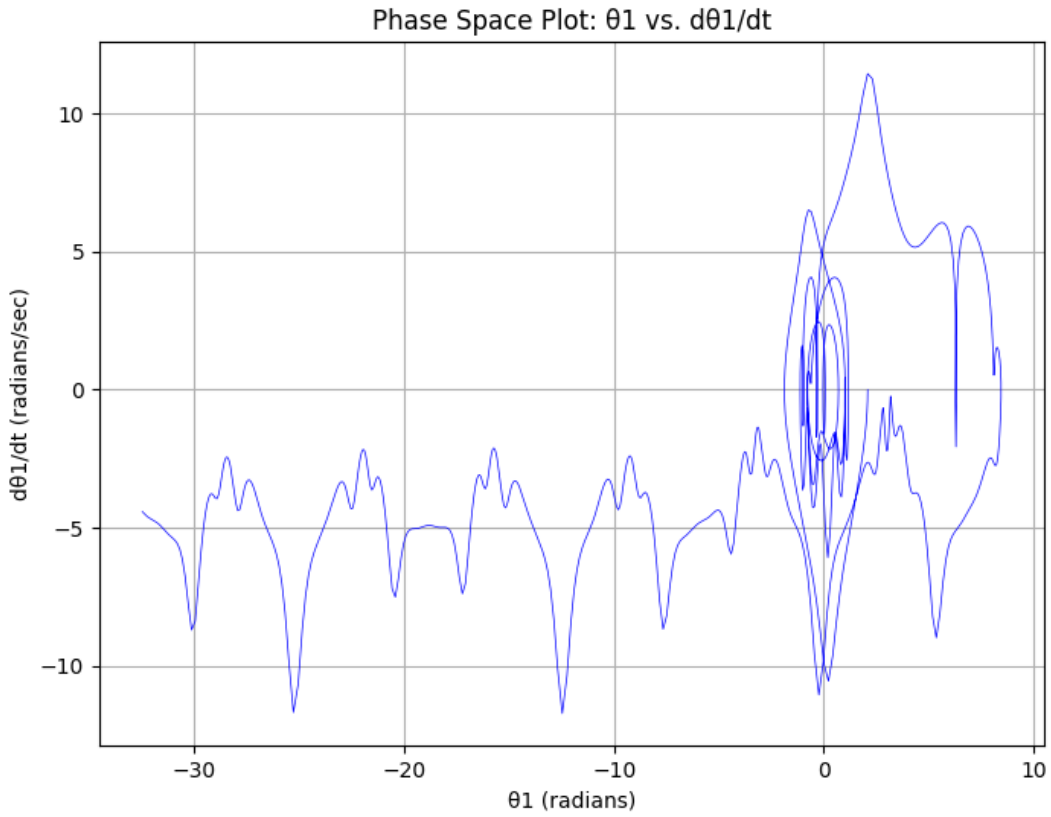
PREDSTAVITEV GIBANJA?



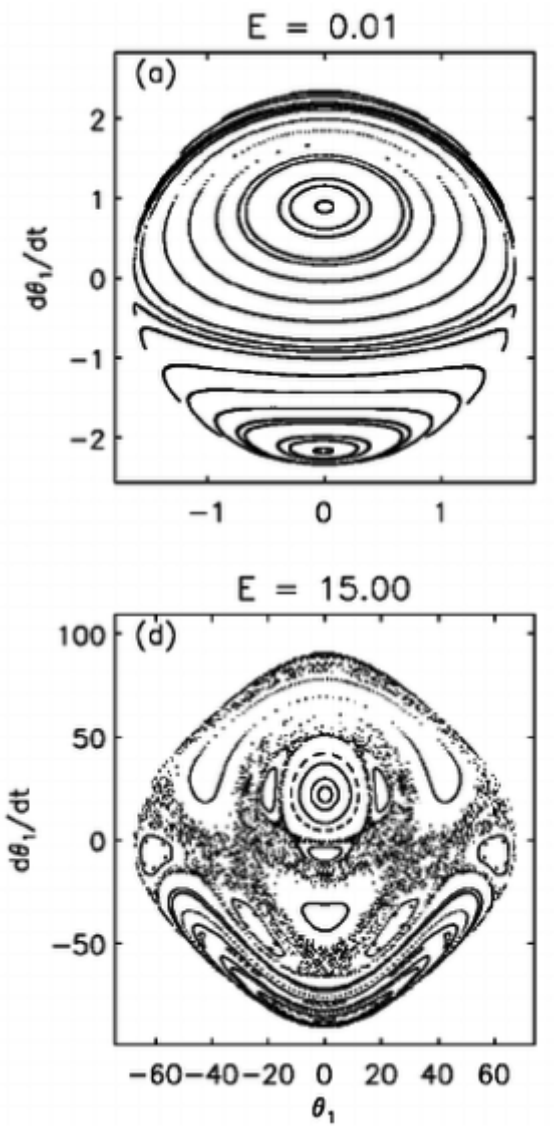


KAOS

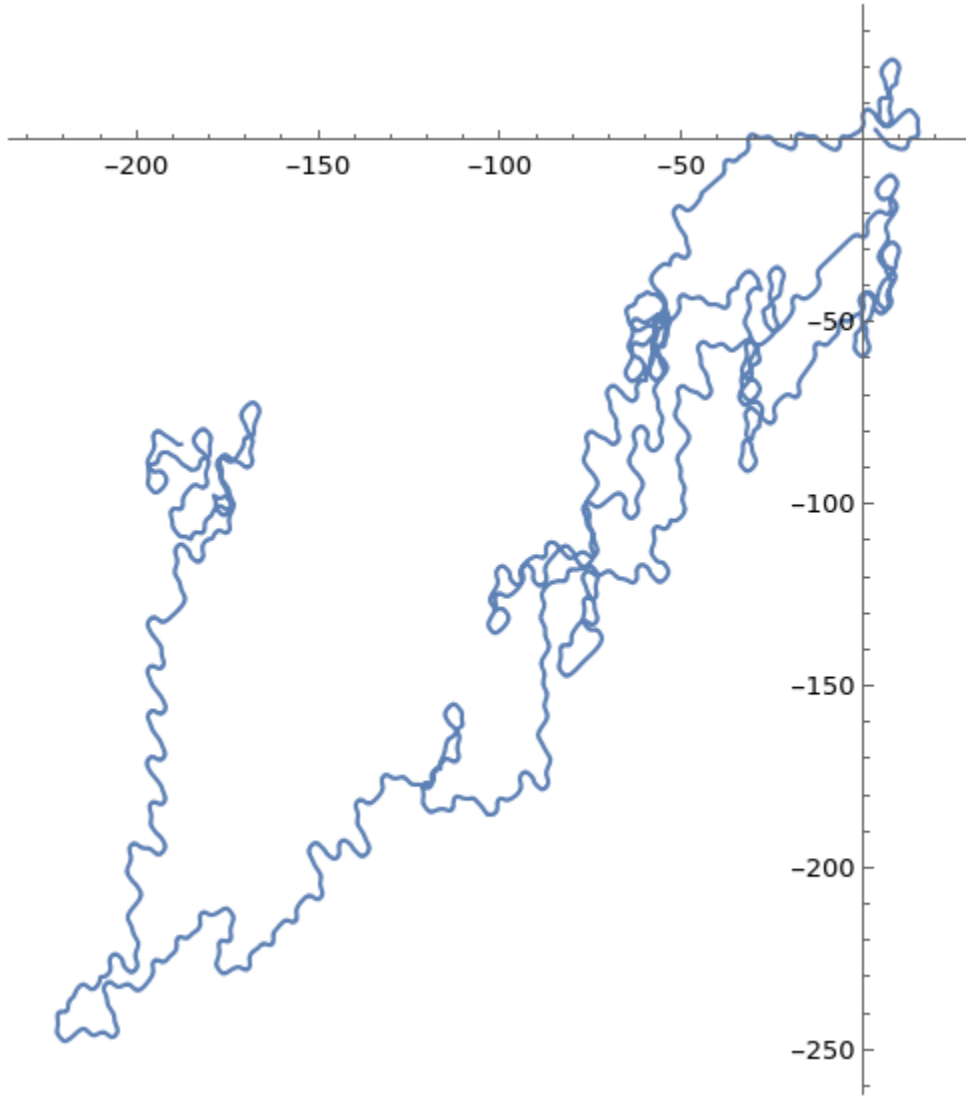
- kaos = občutljivost na začetne pogoje + nepredvidljiv dolgoročni potek
- tudi s popolnimi enačbami napovedovanje po kratkem času odpove
- Lyapunov eksponent meri, kako hitro se trajektorije razhajajo - pozitiven za kaos



POINCAREJEVI ZEMLJEVIDI

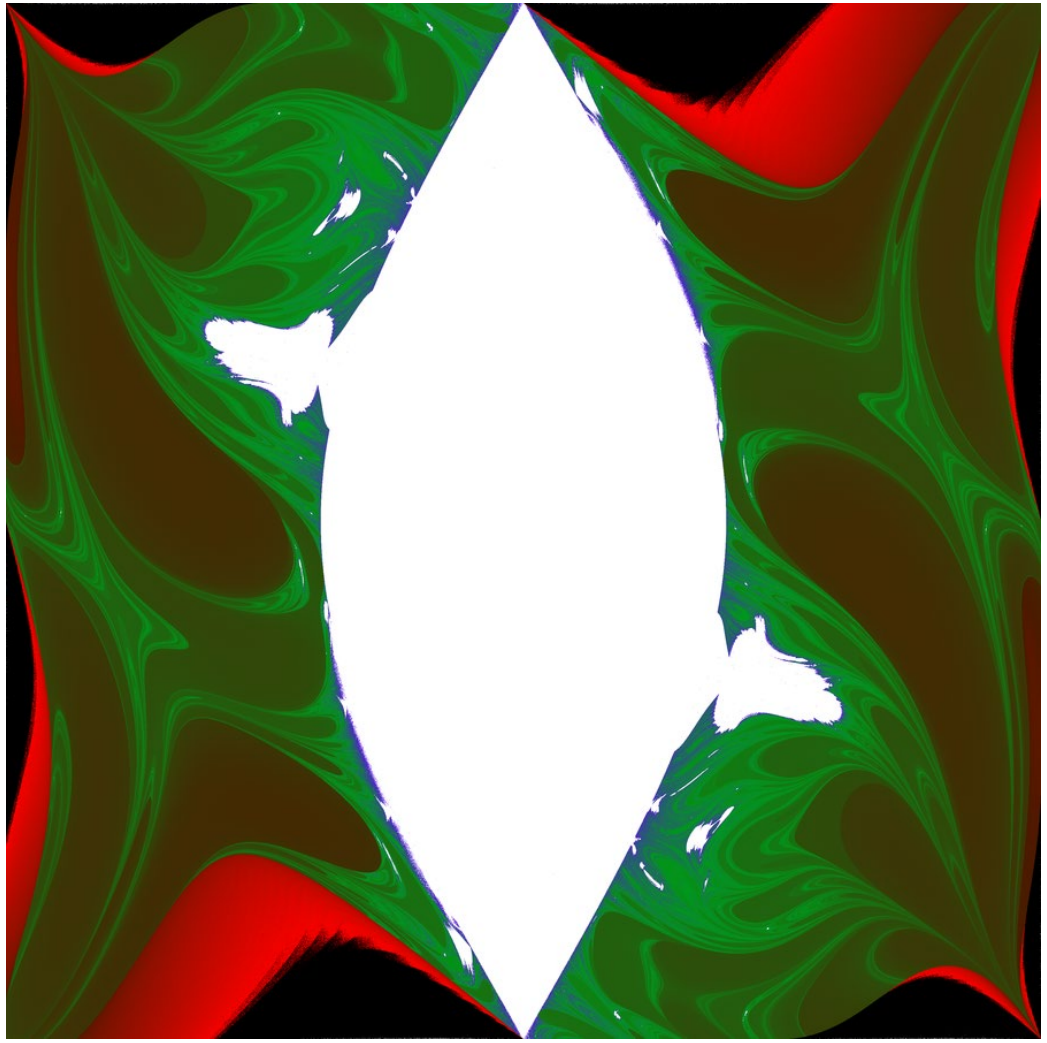


DIFUZIJA



By Grondilu - Own work, CC0, https://commons.wikimedia.org/w/index.php?curid=140628516

FRAKTALI

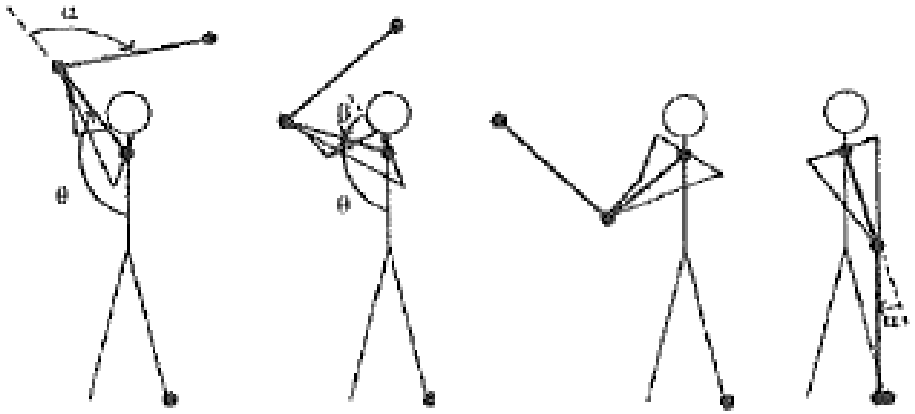


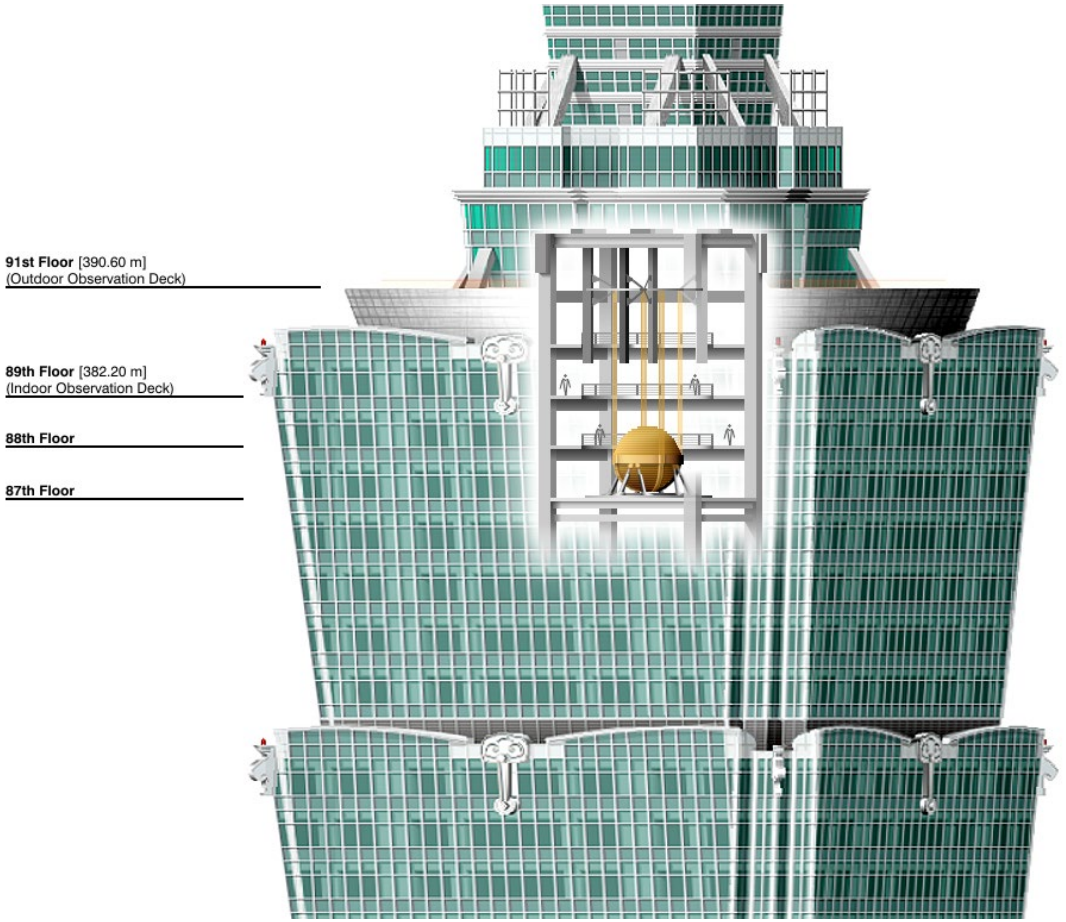
FurtadoG, CC https://commons.wikimedia.org/wiki/File:Double_pendulum_flip_time_2021.png

VIZUALIZACIJA KAOSA

- Začne se urejeno, nato postane nepredvidljivo - obračanje, zanka
- za 0,01 ° različni začetni pogoji se razidejo v sekundah.

UPORABA





Someformofhuman, CC BY-SA 4.0

SIMULACIJE

- https://www.myphysicslab.com/pendulum/double-pendulum-en.html